Problema 267, proposto por Laurentiu Modan, Universidade de Bucarest, Romanía.

Sexa $Z=(X,Y)$ unha variable aleatoria bidimensional, con

$$P\left(\left\{w/X(w)=i,Y(w)=j\right\}\right)=\frac{i}{10\cdot 2^i}\cdot C_{i,j}.$$

con $1\le i\le 4$ e $0\le j\le i$. Estudar se

$$P\left(\left\{w/\left|X(w)-3\right|<2\right\}\right)<\frac{1}{3}.$$

Solución enviada por Bruno Salgueiro Fanego, Viveiro, Lugo.

A función de masa de probabilidade da variable aleatoria Z vén dada por

(i,j)	$(1,0)$	$(1,1)$	$(2,0)$	$(2,1)$	$(2,2)$	$(3,0)$	$(3,1)$	$(3,2)$	$(3,3)$	$(4,0)$	$(4,1)$	$(4,2)$	$(4,3)$	$(4,4)$
$\frac{i\cdot C_{i,j}}{10\cdot 2^i}$	$\frac{4}{80}$	$\frac{4}{80}$	$\frac{4}{80}$	$\frac{8}{80}$	$\frac{4}{80}$	$\frac{3}{80}$	$\frac{9}{80}$	$\frac{9}{80}$	$\frac{3}{80}$	$\frac{2}{80}$	$\frac{8}{80}$	$\frac{12}{80}$	$\frac{8}{80}$	$\frac{2}{80}$

Notemos que está ben definida, pois

$$\sum_{\substack{1\le i\le 4\\ 0\le j\le i}}\frac{i}{10\cdot 2^i}\cdot C_{i,j}=\frac{1}{80}\cdot(4+4+4+8+4+3+9+9+3+2+8+12+8+2)=1.$$

Polo tanto,

$$P\left(\left\{w/\left|X(w)-3\right|<2\right\}\right)=P\left(\left\{w/-2<X(w)-3<2\right\}\right)=P\left(\left\{w/1<X(w)<5\right\}\right)$$

$$=P\left(\left\{w/2\le X(w)\le 4\right\}\right)=1-P\left(\left\{w/X(w)=1\right\}\right)$$

$$=1-\left[P\left(\left\{w/Z(w)=(1,0)\right\}\dot{\cup}\left\{w/Z(w)=(1,1)\right\}\right)\right]$$

$$=1-P\left(\left\{w/Z(w)=(1,0)\right\}\right)-P\left(\left\{w/Z(w)=(1,1)\right\}\right)=1-\frac{1}{20}-\frac{1}{20}=\frac{9}{10}>\frac{1}{3}.$$