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NM53_1: Probar que si $a^2 + b^2 + c^2$, entonces $(a - b)^2 + (b - c)^2 + (c - a)^2 \leq 3$.

NM53_1 – Solution:

We have $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$, but $a^2 + b^2 + c^2 = 1$, so

$$\left.\begin{array}{l}(a + b + c)^2 = 1 + 2(ab + bc + ca) \\ (a + b + c)^2 \geq 0\end{array}\right\} \Rightarrow 1 + 2(ab + bc + ca) \geq 0 \Leftrightarrow 2(ab + bc + ca) \geq -1 \textbf{(1)}$$

$$(a - b)^2 + (b - c)^2 + (c - a)^2 = 2(a^2 + b^2 + c^2) - 2(ab + bc + ca)$$

$$= 2 - 2(ab + bc + ca) \qquad \textbf{(2)}$$

Suming (1) and (2) we obtain $(a - b)^2 + (b - c)^2 + (c - a)^2 \leq 2 - (-1) = 3$, and the problem is solved.

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NM53_2: Probar quenoexistenenterospositivos m y n talesque2(m^2 + mn + n^2)sea un cuadradoperfecto.

NM53_2 – Solution:

Wesupposethat 2(m^2 + mn + n^2) is a perfect square.

Let p be a natural numbersuchthat 2(m^2 + mn + n^2) = p^2.

Wehave 2 | p^2⇔ 2^{2k} | p^2 , where k ϵ N^*, so 2^{2k} | 2(m^2 + mn + n^2) ⇔ 2^{2k-1} | (m^2 + mn + n^2)**(1)**

2k – 1 > 0 => 2 | m^2 + mn + n^2, relationpossibleonlyif m and n are botheven. In this case, theexponent of 2 in thenumbers m^2, mn, n^2iseven, sothe exponent of 2 in (m^2 + mn + n^2) iseven, contradictionwithrelation**(1)** =>the assumptionis false => 2(m^2 + mn + n^2) cannotbe a perfect square.

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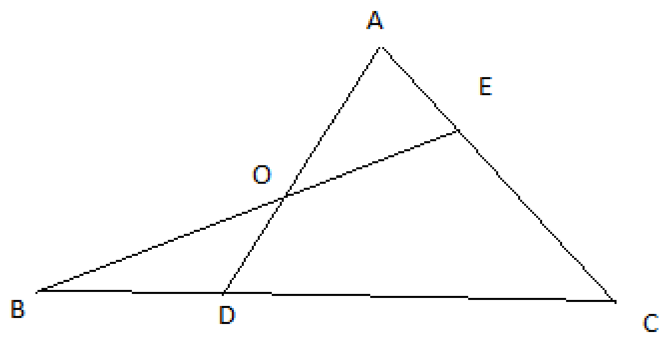
NM53_3: En el triangulo ADC, E es un punto del lado AC, O es un punto del lado AD y la recta EO a la recta DC en un punto B mas alla de D. Se sabe que BD/DC = 4/7 y que AE/EC = 2/3. Calcular AO/OD.

NM53_3 – Solution:



In triangle ABC we have the transversal B-O-E => (Menelaus)

$\frac{AO}{DO} \bullet \frac{DB}{CB} \bullet \frac{CE}{AE} = 1$

From the hypothesis we have $\frac{BD}{DC} = \frac{4}{7} \Leftrightarrow \frac{DB}{CB} = \frac{4}{11}$ and $\frac{CE}{AE} = \frac{3}{2}$, so $\frac{AO}{DO} \bullet \frac{12}{22} = 1$ $\Rightarrow \frac{AO}{DO} = \frac{11}{6}$.

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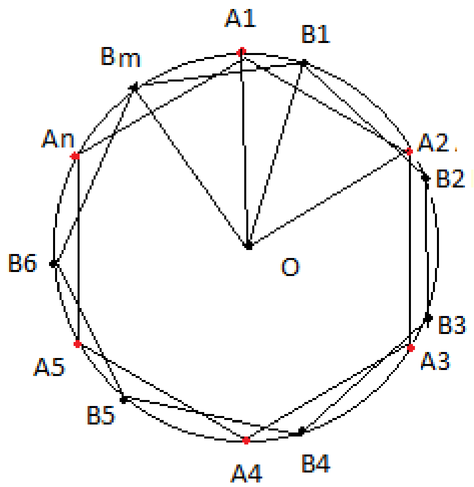
NM53_4: Dos poligons regulares de m y n lados, respectivamente, estan inscritos en la misma circunferencia, La razon de sus areas es m/n. Hallar todos los posibles valores de m y n.

NM53_4 – Solution:



$OA_i = OB_j = R, i = \overline{1,n}\ , j = \overline{1,m}.$

$m(<A_1OA_2) = \frac{360°}{n},\ m(<B_1OB_2) = \frac{360°}{m}.$

Using the cos theorem we obtain:

$$\left.\begin{array}{l} A_iA_{i+1} = \frac{OA_i \cdot OA_{i+1} \cdot \sin\frac{360°}{n}}{2} \\ B_jB_{j+1} = \frac{OB_j \cdot OB_{j+1} \cdot \sin\frac{360°}{m}}{2} \end{array}\right\} \Rightarrow \frac{m}{m} = \frac{m \cdot sin\frac{360°}{n}}{n \cdot sin\frac{360°}{m}} \quad \Rightarrow$$

⇨ $sin\frac{360°}{m} = sin\frac{360°}{n}$ =>we have two cases:

1) m = n, we have infinite solutions
2) $\frac{360°}{m} + \frac{360°}{n} = 180° \Leftrightarrow \frac{1}{n} + \frac{1}{m} = \frac{1}{2}$ => mn = 2m + 2n.

⇔ mn – 2m – 2n + 4 = 4 ⇔ (m – 2)(n – 2) = 4. =>
the solution pairs (m,n) are: (3, 6), (4, 4), (6, 3).

In conclusion, pairs (m,n) that meet the problem requirements are: (k,k), (3,6), (6,3), where $k \in N$, $k \geq 3$.