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PMJ53_1: Un numero natural A lo llamanos ”super 3” si la suma de sus cifras e stres veces mayor que la suma de las cifras del numero A + 1.

Hallar todos los numeros ”super 3” que tienen a lo sumo 4 cifras.

PMJ53_1 – Solution:

Because the sum of digits of A is divisible by 3, A must be divisible by 3 too. This sum is bigger than the sum of digits of A + 1, so the last digit of A is 9.

Let us note s(n) the sum of digits of n.

We have 4 cases:

a) A has only one digit => A = 9 => s(A) = 9.

A + 1 = 10 => s(A + 1) = 1 => s(A) = 9s(A + 1), it is not a solution

b) A has two digits => A € {39, 69, 99}.
A = 39 => s(A) = 12, s(A + 1) = 4 => s(A) = 3s(A + 1) => A = 39 is a solution.
A = 69 => s(A) = 15, s(A + 1) = 7, it is not a solution.
A = 99 => s(A) = 18, s(A + 1) = 1, it is not a solution.

c) A has three digits
   a. The second digit is not 9 => s(A) – s(A + 1) = 8
   s(A + 1) + 8 = 3s(A + 1) => s(A + 1) = 4 => A + 1 € {130, 220, 310, 400} ⬄ A € {129, 219, 309, 399}, but the second digit of 399 is 9.
   b. The second digit is 9 => s(A) – s(A + 1) = 17 => s(A + 1) + 17 = 3s(A + 1) => 2s(A + 1) = 17, contradiction.
   c. A = 999 => A + 1 = 1000 => s(A) = 27s(A + 1), it is not a solution.

d) A has four digits
   a. The third digit is not 9 =>s(A) – s(A + 1) = 8
   s(A + 1) + 8 = 3s(A + 1) => s(A + 1) = 4 => A + 1 € {1030, 1120, 1210, 1300, 2110, 2020, 2200, 3010, 3100, 4000}⬄ A €{1029, 1119, 1209, 1299, 2109, 3009, 3099, 2009, 2199, 3999} but 1299, 3099, 2199, 3999 are not solutions.
   b. The third digit is 9

b.1. The second digit is not 9 => $s(A) - s(A + 1) = 17$ => $s(A + 1) + 17 = 3s(A + 1)$ => $2s(A + 1) = 17$, contradiction.

b.2. The second digit is 9 => $s(A) - s(A + 1) = 26$ => $s(A + 1) = 13$, but A + 1 has last three digits 0, so $s(A + 1) \le 9$ =>contradiction.

c. $A = 9999$ => $A + 1 = 10000$ => $s(A) = 9999s(A + 1)$, it is not a solution

In conclusion, the solutions are: 39, 129, 219, 309, 1029, 1119, 1209, 2109, 3009, 2009.

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PMJ53_2: Demonstrar que la fraction

$$\frac{2013^{2013}-2013^{2012}-2013^{2011}}{2012^{2013}-2012^{2012}-2012^{2011}-2013}$$

No es irreductible.

PMJ53_2 – Solution:

$2013^{2013} - 2013^{2012} - 2013^{2011} = 2013^{2011}(2013^2 - 2013 - 1)$

Let us note u(n) the last digit of the natural number n.

$u(2013^2 - 2013 - 1) = u(9 - 3 - 1) = 5 \Rightarrow 5 \mid 2013^{2013} - 2013^{2012} - 2013^{2011}$ **(1)**

$2012^{2013} - 2012^{2012} - 2012^{2011} - 2013 = 2012^{2011}(2012^2 - 2012 - 1)$

$u(2012^2 - 2012 - 1) = u(4 - 2 - 1) = 1 \Rightarrow u(2012^{2011}(2012^2 - 2012 - 1)) = u(2012^{2011}) = 8 \Rightarrow u(2012^{2013} - 2012^{2012} - 2012^{2011} - 2013) = u(8 - 3) = 5$
$\Rightarrow 5 \mid 2012^{2013} - 2012^{2012} - 2012^{2011} - 2013$ **(2)**

From (1) and (2) we obtain that the fraction is reductible by 5.

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PMJ53_3:

a) Descompuner el numero 2012 en suma de numeros naturales consecutivos
b) Descompuner el numero 2012 en suma de numeros naturales pares consecutivos
c) Descompuner el numero 2012 en suma de numeros naturales impares consecutivos

PMJ53_3 – Solution:

a) 2012 = 248 + 249 + 250 + 251 + 252 + 253 + 254 + 255.
b) 2012 = 500 + 502 + 504 + 506.
c) 2012 = 1005 + 1007.

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PMJ53_4: El area de un rectangulo es $2m^2$. Si se aumenta la longitud y la anchura en 2m, el area aumenta en $8m^2$. Hallar el perimetro del rectangulo inicial.

PMJ53_4 – Solution:

Let us note L the length and l the width of the rectangle.

$L \cdot l = 2m^2$.

$(L + 2m)(l + 2m) = 2 + 8 = 10m^2 \Leftrightarrow L \cdot l + 4m^2 + 2m \cdot (L + l) = 10m^2 \Leftrightarrow 2m \cdot (L + l) = 4m^2 \Leftrightarrow L + l = 2m$.

$(L + l)^2 = 4m^2 \Leftrightarrow L^2 + l^2 + 2L \cdot l = 4m^2 \Leftrightarrow L^2 + l^2 = 4m^2 - 4m^2 = 0$, but $L^2 > 0$ and $l^2 > 0 \Rightarrow L = l = 0$, contradiction.

In conclusion, the respectively rectangle cannot exist.